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A current–current energy–momentum tensor for chiral dynamics

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Abstract. Starting from a canonical quantum field theory model and imposing invariance of the theory under a group, acting nonlinearly on the fields, a group-invariant energy–momentum tensor of the Sugawara form is constructed which is free of ambiguities due to factor ordering and has the required properties.

1. Introduction

In order to quantize a system with derivative coupling such as occurs for example in chiral theories or gravitational theories, the ordering of noncommuting variables has to be taken into account.

Charap (1973) has shown for a quantum-mechanical model of a system of massless, pseudoscalar mesons, that imposing chiral invariance is sufficient to completely remove the ambiguities associated with the ordering of factors. Further, the resulting hamiltonian is unique and of the charge–charge form. This suggests that the corresponding quantum field theory might have a canonical energy–momentum tensor of the current–current form as in Sugawara's (1968) field theory of currents.

In § 2, the classical field theory is briefly discussed and then in the next section the corresponding quantum field theory. Invariance of the system under a group G acting nonlinearly on the fields is imposed in § 4 and this requirement is sufficient to remove the factor-ordering ambiguities. The canonical energy–momentum tensor $T^{\mu\nu}$ is discussed in the final section. It is unique, group invariant and of the current–current form. Furthermore, it is locally conserved, satisfies the condition of Schwinger (1963) and the generators of space–time translations and rotations constructed from $T^{\mu\nu}$ satisfy the Poincaré algebra.

Unlike in the original Sugawara model, there is no parity doubling, but the difficulties of interpretation due to products of operators at the same space–time point are still present.

2. Classical field theory

Consider the lagrangian density $L(\phi, \partial_\mu\phi)$ of the form

$$L = \tfrac{1}{2}\partial_\mu\phi^\alpha(x)g_{\alpha\beta}(\phi(x))\partial^\mu\phi^\beta(x) \tag{2.1}$$

where the $g_{\alpha\beta}$ are, as yet unspecified, functions of the multiplet of spinless fields $\phi^\alpha(x)$ ($\alpha = 1, 2, \ldots m$; $x \equiv x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (t, \boldsymbol{x})$), and without loss of generality the matrix $g_{\alpha\beta}$ may be taken symmetrical.

Each field $\phi^\alpha(x)$ may be associated with a Lorentz four-vector $\pi^\mu_\alpha(x)$ given by

$$\pi^\mu_\alpha(x) \equiv \partial L/\partial(\partial_\mu \phi^\alpha(x)) \tag{2.2}$$

which, for the lagrangian density (2.1), gives

$$\pi^\mu_\alpha = g_{\alpha\beta}\partial^\mu \phi^\beta. \tag{2.3}$$

In order to construct a hamiltonian density $H(\phi, \nabla\phi, \pi, \nabla\pi)$ (2.2) must be solvable for $\partial^0\phi^\alpha$ as a function of $\pi_\alpha \equiv \pi^0_\alpha$, which from (2.3) means the determinant of $g_{\alpha\beta}$ must be nonzero, or equivalently that the symmetric differential form

$$\mathrm{d}s^2 = g_{\alpha\beta}\, \mathrm{d}\phi^\alpha\, \mathrm{d}\phi^\beta \tag{2.4}$$

defines a pseudo-riemannian metric on the manifold M, parametrized by the fields ϕ^α. It is further assumed that this form is positive definite and so defines a riemannian metric.

Now consider the action of a compact, semi-simple Lie group G, with generators F^a ($a = 1, 2, \ldots n$), on the fields ϕ^α. Then associated with the infinitesimal local transformation

$$\phi^\alpha(x) \to \phi^\alpha(x) + Q_a f^{a\alpha}(\phi) \tag{2.5}$$

of the fields, are the Noether current densities $\mathcal{J}^{a\mu}(x)$ with

$$\mathcal{J}^{a\mu} = -\pi^\mu_\alpha f^{a\alpha}. \tag{2.6}$$

The Q_a are infinitesimal gauge parameters and the $f^{a\alpha}(\phi)$ some functions of the fields, which satisfy the group laws.

If, further, G is an invariance group of the system, the current densities are conserved and the form (2.4) becomes a group invariant metric on the manifold M, satisfying Killing's equations, which may be written

$$f^{\alpha\beta}{}_{,\gamma} g^{\alpha\gamma} + f^{a\alpha}{}_{,\gamma} g^{\beta\gamma} - f^{a\gamma} g^{\alpha\beta}{}_{,\gamma} = 0 \tag{2.7a}$$

or equivalently

$$f^{a\alpha} g_{\beta\gamma,\alpha} + f^{a\alpha}{}_{,\beta} g_{\alpha\gamma} + f^{a\alpha}{}_{,\gamma} g_{\alpha\beta} = 0 \tag{2.7b}$$

where $\|g^{\alpha\beta}\|$ is the matrix inverse of $\|g_{\alpha\beta}\|$ and $f^{\alpha\beta}{}_{,\gamma}$ denotes $\partial f^{\alpha\beta}/\partial\phi^\gamma$.

If the action of the group G on the manifold M is linear, then Killing's equation (2.7a) implies that $g^{\alpha\beta}$ is a second-rank group tensor. However, when the action of the group is nonlinear, the solutions $g^{\alpha\beta}$ are

$$g^{\alpha\beta} = \nu\, e_{ab} f^{a\alpha} f^{b\beta} \tag{2.8a}$$

where ν is an arbitrary constant and $\|e_{ab}\|$ is the matrix inverse of $\|e^{ab}\|$ given by

$$e^{ab} = f^{ac}{}_d f^{bd}{}_c \tag{2.8b}$$

(in a particular basis, the canonical Cartan basis, $e_{ab} \sim \delta_{ab}$). The $f^{ab}{}_c$ are the structure constants of the Lie algebra of the group G.

The canonical energy–momentum tensor density $T^{\mu\nu}$, defined for spinless fields as

$$T^{\mu\nu} \equiv \pi^\mu_\alpha \partial^\nu \phi^\alpha - \eta^{\mu\nu} L \tag{2.9}$$

where $\eta^{\mu\nu}$ is the space–time metric with signature $(1, -1, -1, -1)$, may now be written

$$T^{\mu\nu} = v\, e_{ab}\mathscr{J}^{a\kappa}\mathscr{J}^{b\lambda}(\delta^{\mu}_{\kappa}\delta^{\nu}_{\lambda} - \tfrac{1}{2}\eta^{\mu\nu}\eta_{\kappa\lambda}). \tag{2.10}$$

Finally, the equations of motion are

$$\partial_{\mu}\pi^{\mu}_{\alpha} = -\tfrac{1}{2}\eta_{\mu\nu}g^{\beta\gamma}{}_{,\alpha}\pi^{\mu}_{\beta}\pi^{\nu}_{\gamma} \tag{2.11a}$$

or

$$\Box\phi^{\beta} + \Gamma^{\beta}_{\alpha\gamma}\partial_{\mu}\phi^{\alpha}\partial^{\mu}\phi^{\gamma} = 0 \tag{2.11b}$$

with the Christoffel symbol $\Gamma^{\beta}_{\alpha\gamma}$ in (2.11b) defined by

$$\Gamma^{\beta}_{\alpha\gamma} \equiv \tfrac{1}{2}g^{\delta\beta}(g_{\delta\alpha,\gamma} + g_{\delta\gamma,\alpha} - g_{\alpha\gamma,\delta}).$$

3. Quantum field theory

To obtain the quantum analogue of the classical system described in the previous section, the canonical equal-time commutation relations

$$[\phi^{\alpha}(x), \phi^{\beta}(y)] = 0 \tag{3.1a}$$

$$[\pi_{\alpha}(x), \pi_{\beta}(y)] = 0 \tag{3.1b}$$

$$[\phi^{\alpha}(x), \pi_{\beta}(y)] = \mathrm{i}\delta^{\alpha}_{\beta}\delta^{3}(\boldsymbol{x} - \boldsymbol{y}) \tag{3.1c}$$

are imposed, where $\phi^{\alpha}(x)$, $\pi_{\beta}(y)$ are now hermitian operators.

The dynamics is specified by giving the form of the hamiltonian density operator. This is formally taken over from the classical theory, except that now the ordering of the noncommuting factors has to be taken into account. Thus H is taken to be of the form

$$H = \tfrac{1}{8}\{\pi_{\alpha}, \{\pi_{\beta}, g^{\alpha\beta}\}\} + \tfrac{1}{2}\{\pi_{\alpha}, u^{\alpha}\} + v + \tfrac{1}{2}g_{\alpha\beta}\partial_{k}\phi^{\alpha}\partial^{k}\phi^{\beta} \tag{3.2}$$

where k takes the values 1, 2, 3. In (3.2) all possible orderings have been allowed for and the u^{α} and v are arbitrary functions of the fields ϕ^{α}. If u^{α} and v are real functions, H is hermitian and furthermore if $g^{\alpha\beta}$ is the same function of the fields as in the classical case, and if u^{α} and v tend to zero with Planck's constant then H is a suitable quantum analogue of the corresponding classical hamiltonian density H_{c}

$$H_{\mathrm{c}} = \tfrac{1}{2}(\pi_{\alpha}g^{\alpha\beta}\pi_{\beta} + g_{\alpha\beta}\partial_{k}\phi^{\alpha}\partial^{k}\phi^{\beta}).$$

The Heisenberg equations of motion are

$$\partial^{0}\phi^{\alpha} = \tfrac{1}{2}\{\pi_{\beta}, g^{\alpha\beta}\} + u^{\alpha} \tag{3.3}$$

and

$$\partial^{0}\pi_{\alpha} = -\tfrac{1}{8}\{\pi_{\beta}, \{\pi_{\gamma}, g^{\beta\gamma}{}_{,\alpha}\}\} - \tfrac{1}{2}\{\pi_{\beta}, u^{\beta}{}_{,\alpha}\} - v_{,\alpha} - \tfrac{1}{2}(g_{\alpha\gamma,\beta} + g_{\beta\alpha,\gamma} - g_{\gamma\beta,\alpha})\,\partial_{k}\phi^{\beta}\partial^{k}\phi^{\gamma}.$$

Inverting (3.3) gives

$$\pi_{\alpha} = \tfrac{1}{2}\{g_{\alpha\beta}, \partial^{0}\phi^{\beta}\} - u_{\alpha} \tag{3.4}$$

where

$$u_{\alpha} \equiv g_{\alpha\beta}u^{\beta}. \tag{3.5}$$

Consider now the action of a compact, semi-simple group G on the fields ϕ^α. Thus, there is a set of generators F^a, with the algebra

$$[F^a, F^b] = \mathrm{i} f^{ab}{}_c F^c \tag{3.6}$$

where the $f^{ab}{}_c$ are the c number structure constants of the algebra and the generators are assumed to arise from local current densities $\mathscr{J}^{a0}(x)$ (see beginning of § 4)

$$F^a(t) = \int \mathrm{d}^3\boldsymbol{x}\mathscr{J}^{a0}(x). \tag{3.7}$$

Following the same procedure as for constructing the hamiltonian density (3.2), $\mathscr{J}^{a0}(x)$ is taken to have the form

$$\mathscr{J}^{a0} = -\tfrac{1}{2}\{\pi_\alpha, f^{a\alpha}\} + l^a \tag{3.8}$$

where the l^a are some functions of the fields which tend to zero with Planck's constant.

The action of the group on the fields ϕ^α and conjugate momenta π_α can now be determined. From (3.1) and (3.8)

$$[\mathscr{J}^{a0}(x), \phi^\alpha(y)] = \mathrm{i} f^{a\alpha}(\phi)\delta^3(\boldsymbol{x}-\boldsymbol{y})$$

and using (3.7)

$$[F^a, \phi^\alpha(y)] = \mathrm{i} f^{a\alpha}(\phi). \tag{3.9}$$

The compatibility of (3.6) and (3.9) is embodied in the Jacobi identity for F^a, F^b and ϕ^α which gives

$$f^{a\beta} f^{b\alpha}{}_{,\beta} - f^{b\beta} f^{a\alpha}{}_{,\beta} = f^{ab}{}_c f^{c\alpha}. \tag{3.10}$$

Similarly

$$[F^a, \pi_\beta(y)] = -\tfrac{1}{2}\mathrm{i}\{\pi_\alpha, f^{a\alpha}{}_{,\beta}\} + \mathrm{i}\, l^a{}_{,\beta}$$

so that the group action is linear on the momenta π_α. The Jacobi identity for F^a, F^b and π_γ now implies

$$f^{a\alpha} l^b{}_{,\alpha} - f^{b\alpha} l^a{}_{,\alpha} = f^{ab}{}_c l^c. \tag{3.11}$$

Also from (3.8) and (3.11)

$$[\mathscr{J}^{a0}(x), \mathscr{J}^{b0}(y)] = \mathrm{i} f^{ab}{}_c \mathscr{J}^{c0}\delta^3(\boldsymbol{x}-\boldsymbol{y}) \tag{3.12}$$

which is compatible with (3.6).

4. Invariance

The system is now assumed to be invariant under the action of the group G

$$[F^a, P^\mu] = 0 \tag{4.1}$$

where the P^μ are the space–time translation operators. Thus F^a has the form (3.7) and the associated Lorentz four-vector current densities $\mathscr{J}^{a\mu}(x)$ are locally conserved. Imposing this latter condition gives the requirement

$$[H(x), \mathscr{J}^{a0}(y)] = -\mathrm{i}\mathscr{J}^{ak}(x)\partial_k^x\delta^3(\boldsymbol{x}-\boldsymbol{y}) \tag{4.2}$$

where ∂_k^x denotes differentiation with respect to the argument x in the delta function.

Explicit calculation of the left-hand side of (4.2) using (3.2) and (3.8) gives

$$[H(x), \mathcal{J}^{a0}(y)] = -\mathrm{i}\delta^3(\boldsymbol{x}-\boldsymbol{y})[\tfrac{1}{8}\{\pi_\alpha, \{\pi_\beta, K^{a\alpha\beta}\}\} + \tfrac{1}{2}\{\pi_\alpha, K^{a\alpha}\} + K^a]$$
$$+\mathrm{i}f^{a\alpha}g_{\alpha\beta}\partial^k\phi^\beta\partial_k^x\delta^3(\boldsymbol{x}-\boldsymbol{y}) \tag{4.3}$$

where

$$K^{a\alpha\beta} = f^{a\beta}{}_{,\gamma}g^{\alpha\gamma} + f^{a\alpha}{}_{,\gamma}g^{\beta\gamma} - f^{a\gamma}g^{\alpha\beta}{}_{,\gamma} \tag{4.4a}$$

$$K^{a\alpha} = f^{a\alpha}{}_{,\beta}u^\beta - f^{a\beta}u^\alpha{}_{,\beta} - g^{\alpha\beta}l^a{}_{,\beta} \tag{4.4b}$$

$$K^a = \tfrac{1}{8}f^{a\gamma}{}_{,\alpha\beta}g^{\alpha\beta}{}_{,\gamma}(\delta^3(0))^2 - f^{a\alpha}v_{,\alpha} - l^a{}_{,\alpha}u^\alpha. \tag{4.4c}$$

Imposing (4.2) and identifying, by analogy with the classical expression (2.6),

$$\mathcal{J}^{ak} = -f^{a\alpha}g_{\alpha\beta}\partial^k\phi^\beta \tag{4.5}$$

(see (5.12) for the justification of this) requires all the functions K to vanish.

Solving for each of the equations (4.4*a*), (4.4*b*) and (4.4*c*) in that order for $g^{\alpha\beta}$, u^α and v respectively and assuming the action of G is nonlinear on the fields (see (2.8*a*) and (2.8*b*)) then

$$g^{\alpha\beta} = \nu\, e_{ab}f^{a\alpha}f^{b\beta} \tag{2.8a}$$

$$u^\alpha = -\nu\, e_{ab}f^{a\alpha}l^b \tag{4.6a}$$

$$v = \tfrac{1}{2}\nu\, e_{ab}\{l^a l^b + \tfrac{1}{4}f^{a\alpha}{}_{,\beta}f^{b\beta}{}_{,\alpha}(\delta^3(0))^2\}. \tag{4.6b}$$

5. The energy–momentum tensor

Substituting now the expressions (2.8) and (4.6) for $g^{\alpha\beta}$, u^α and v in that for the hamiltonian density (3.2) and then using (3.8) and (4.5) for $\mathcal{J}^{a0}$ and $\mathcal{J}^{ak}$ respectively, the hamiltonian density may be written

$$T^{00} \equiv H = \tfrac{1}{2}\nu\, e_{ab}(\mathcal{J}^{a0}\mathcal{J}^{b0} + \boldsymbol{\mathcal{J}}^a \cdot \boldsymbol{\mathcal{J}}^b). \tag{5.1}$$

All the operators are hermitian and $\nu\, e_{ab}$ is positive definite so that H is a positive semi-definite operator as required.

An expression for the energy–momentum tensor may now be found from (5.1)

$$T^{\mu\nu} = \tfrac{1}{2}\nu\, e_{ab}\{\mathcal{J}^{a\kappa}, \mathcal{J}^{b\lambda}\}(\delta^\mu_\kappa\,\delta^\nu_\lambda - \tfrac{1}{2}\eta^{\mu\nu}\eta_{\kappa\lambda}) \tag{5.2}$$

and this now has to be shown to have the correct properties, in order to be taken as the energy–momentum tensor.

First, from (5.2) Schwinger's (1963) condition may be seen to be satisfied

$$[T^{00}(x), T^{00}(y)] = -\mathrm{i}(T^{0k}(x) + T^{0k}(y))\partial_k^x\delta^3(\boldsymbol{x}-\boldsymbol{y}). \tag{5.3}$$

Then with the usual definitions

$$P^k = \int \mathrm{d}^3\boldsymbol{x}\,T^{0k}(x) \tag{5.4}$$

$$J^{kl} = \int \mathrm{d}^3\boldsymbol{x}(x^kT^{0l}(x) - x^lT^{0k}(x)) \tag{5.5}$$

the following relations hold:

$$[P^k, \phi^\alpha(y)] = -\mathrm{i}\,\partial^k\phi^\alpha(y)$$

$$[J^{kl}, \phi^\alpha(y)] = -\mathrm{i}(y^k\,\partial^l\phi^\alpha(y) - y^l\,\partial^k\phi^\alpha(y))$$

so that P^k and J^{kl} are the infinitesimal generators for three-dimensional translations and rotations respectively. Also from (5.4) and (5.5)

$$[P^k, P^l] = 0 \tag{5.6a}$$

$$[P^k, J^{lm}] = -\mathrm{i}(\delta^{kl}P^m - \delta^{km}P^l) \tag{5.6b}$$

$$[J^{kl}, J^{mn}] = -\mathrm{i}(\delta^{lm}J^{nk} - \delta^{km}J^{ln} + \delta^{ln}J^{km} - \delta^{kn}J^{ml}). \tag{5.6c}$$

These relations (5.6) together with Schwinger's condition (5.3) then guarantee the Poincaré invariance of the system.

Using Killing's equations (2.7*b*) and the expressions for the currents (3.8) and (4.5) and the identity

$$f(x)\,\partial_y^k\,\delta^3(\boldsymbol{x}-\boldsymbol{y}) = f(y)\,\delta_y^k\,\delta^3(\boldsymbol{x}-\boldsymbol{y}) + \delta^3(\boldsymbol{x}-\boldsymbol{y})\,\partial_y^k f(y) \tag{5.7}$$

the current algebra of the system is

$$[\mathscr{J}^{a0}(x), \mathscr{J}^{b0}(y)] = \mathrm{i}f^{ab}{}_c\mathscr{J}^{c0}\delta^3(\boldsymbol{x}-\boldsymbol{y}) \tag{3.12}$$

$$[\mathscr{J}^{a0}(x), \mathscr{J}^{bk}(y)] = \mathrm{i}f^{ab}{}_c\mathscr{J}^{ck}\delta^3(\boldsymbol{x}-\boldsymbol{y}) - \mathrm{i}S^{ab}(y)\partial_y^k\delta^3(\boldsymbol{x}-\boldsymbol{y}) \tag{5.8}$$

$$[\mathscr{J}^{ak}(x), \mathscr{J}^{bl}(y)] = 0$$

where the q number Schwinger term S^{ab} in (5.8) is given by

$$S^{ab} = f^{a\alpha}g_{\alpha\beta}f^{b\beta}. \tag{5.9}$$

In fact, the algebra of currents and Schwinger terms closes

$$[\mathscr{J}^{ak}(x), S^{bc}(y)] = 0$$

$$[\mathscr{J}^{a0}(x), S^{bc}(y)] = \mathrm{i}(f^{ab}{}_dS^{dc} + f^{ac}{}_dS^{db})\delta^3(\boldsymbol{x}-\boldsymbol{y})$$

where the relation

$$S^{ab}{}_{,\alpha}f^{c\alpha} = f^{ca}{}_dS^{db} + f^{cb}{}_dS^{da}$$

has been used.

Also using in addition equation (3.4*a*) for u^α it is easy to see that

$$\mathscr{J}^{c\mu}(x) = \tfrac{1}{2}ve_{ab}\{S^{ac}(x), \mathscr{J}^{b\mu}(x)\} \tag{5.10}$$

and

$$\partial^\mu\phi^\gamma(x) = -\tfrac{1}{2}ve_{ab}\{\mathscr{J}^{a\mu}(x), f^{b\gamma}(x)\}. \tag{5.11}$$

Inverting (5.11) using (5.10) gives

$$\mathscr{J}^{a\mu} = -\tfrac{1}{2}\{f^{a\alpha}g_{\alpha\beta}, \partial^\mu\phi^\beta\} \tag{5.12}$$

which shows that for the choice of current components (3.8) and (4.5) $\mathscr{J}^{a\mu}(x)$ is a Lorentz four-vector. Direct calculation using (5.12), (3.10) and (2.7*b*) now gives

$$\partial^\mu\mathscr{J}^{e\nu} - \partial^\nu\mathscr{J}^{e\mu} = \tfrac{1}{2}f^e{}_{db}\{\mathscr{J}^{d\nu}, \mathscr{J}^{b\mu}\}. \tag{5.13}$$

Now from the expression (5.2) for $T^{\mu\nu}$

$$\partial_\mu T^{\mu\nu} = \tfrac{1}{2}\nu e_{ab}\{\mathscr{J}^a_\mu, \partial^\mu \mathscr{J}^{b\nu} - \partial^\nu \mathscr{J}^{b\mu}\} \tag{5.14}$$

and substituting (5.13) and using (5.10) and symmetry arguments, the right-hand side of (5.14) vanishes so that $T^{\mu\nu}$ is locally conserved. Thus $T^{\mu\nu}$ as given by (5.2) has the properties required of an energy–momentum tensor. In addition, $T^{\mu\nu}$ is easily checked to be group invariant and the only ambiguity in $T^{\mu\nu}$ once given the transformation law (3.9) is in the functions l^a. However, changing the functions l^a is equivalent to performing the unitary transformation

$$\mathscr{J}^{a0}(x) = \mathrm{e}^{-\mathrm{i}N(t)}\tilde{\mathscr{J}}^{a0}(x)\,\mathrm{e}^{\mathrm{i}N(t)} \tag{5.15}$$

on the solutions $\mathscr{J}^{a0}(x)$ of (3.12) and thus on $T^{\mu\nu}$ as given by (5.2). So, up to a unitary transformation, the group invariant energy–momentum tensor is unique. In (5.15)

$$N(t) = \int \mathrm{d}^3 y u(y)$$

for some function u of the fields, such that $u_{,\alpha} = u_\alpha$ and

$$\tilde{\mathscr{J}}^{a0} = -\tfrac{1}{2}\{\pi_\alpha, f^{a\alpha}\}.$$

This means, in particular that, after a suitable unitary transformation, the hamiltonian density (3.2) may be written

$$H = \tfrac{1}{8}\{\pi_\alpha, \{\pi_\beta, g^{\alpha\beta}\}\} + \tfrac{1}{2}g_{\alpha\beta}\partial_k\phi^\alpha\partial^k\phi^\beta + \tfrac{1}{8}\nu e_{ab}f^{a\alpha}{}_{,\beta}f^{b\beta}{}_{,\alpha}(\delta^3(0))^2.$$

The presence of the final term involving $(\delta^3(0))^2$ has been noted previously by Dowker and Mayes (1971) and Suzuki and Hattori (1972) (see the discussion in Charap (1973) however).

Finally, the parity-doubling objections of Dashen and Frishman (1969) are removed in this model since the Schwinger terms are q numbers and $T^{\mu\nu}_{\mathrm{R}} = T^{\mu\nu}_{\mathrm{L}}$ since

$$e_{ab}\{\mathscr{J}^{a\mu}_{\mathrm{R}}, \mathscr{J}^{b\nu}_{\mathrm{R}}\} = e_{ab}\{\mathscr{J}^{a\mu}_{\mathrm{L}}, \mathscr{J}^{b\nu}_{\mathrm{L}}\}$$

where the suffixes R and L refer to T being expressed entirely in terms of right $\mathscr{J}^{a\mu}_{\mathrm{R}}(x)$ or left $\mathscr{J}^{a\mu}_{\mathrm{L}}(x)$ chiral currents respectively and defined as one half the sum and one half the difference of the usual vector and axial vector currents.

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